Exercise 1: Write down in full the following expressions ($(i, j, k = 1,2,3)$.

(i) σ_{ii} , (ii) $A_{ij}\, B_{ij}$, (iii) $\sigma_{ij} = \lambda\, \varepsilon_{kk}\, \delta_{ij} + 2\, \mu\, \varepsilon_{ij}$, (iv) $\dfrac{\partial^2 F}{\partial x_i\, \partial x_i}$, (v) $\dfrac{\partial \sigma_{ij}}{\partial x_j} + b_i = 0$.

Exercise 2: Evaluate the following expressions when $(i, j, k = 1,2,3)$

(i) δ_{ii} , (ii) $\delta_{ij}\, \delta_{ij}$, (iii) $\delta_{ij}\, A_{ik}$.

Exercise 3: Show that $\left(P_{ijk} + P_{jik} + P_{ikj}\right) x_i\, x_j\, x_k = 3\, P_{ijk}\, x_i\, x_j\, x_k$.

Exercise 4: For a solid of volume V and surface area $\partial\Omega$, use the divergence theorem to show,

$$\int_{\partial\Omega} x_i n_j dS = \delta_{ij} V .$$

Exercise 5: For the vector, $\boldsymbol{b} = \nabla \times \boldsymbol{u}$ show that,

$\int_{\partial\Omega} \lambda b_i n_i dS = \int_{\Omega} \lambda_{,i} b_i dV$ where $\lambda(x_i)$ is a scalar function of space variables.

Exercise 6: Show the following identities,

(a) $\boldsymbol{u} \cdot \boldsymbol{L}^T \boldsymbol{v} = \boldsymbol{L}\boldsymbol{u} \cdot \boldsymbol{v} = \boldsymbol{v} \cdot \boldsymbol{L}\boldsymbol{u}$
(b) $(\boldsymbol{u} \otimes \boldsymbol{v})(\boldsymbol{a} \otimes \boldsymbol{b}) = (\boldsymbol{v} \cdot \boldsymbol{a})\boldsymbol{u} \otimes \boldsymbol{b} = \boldsymbol{u} \otimes \boldsymbol{b}(\boldsymbol{v} \cdot \boldsymbol{a})$

Exercise 7: If the tensor $\boldsymbol{A}$ is symmetric show that $A_{ij} L_{ij} = A_{ij} L^s_{ij}$, where L^s_{ij} are the components of the the symmetric tensor $\boldsymbol{L}^{\mathbf{s}}$ of $\boldsymbol{L}$.

Exercise 8: Show that the quantratic form,

$$\boldsymbol{x} \cdot \boldsymbol{L}\boldsymbol{x} = L_{ij} x_i x_j$$

remains unchanged if $\boldsymbol{L}$ is replaced by its symmetric part $\boldsymbol{L}^{\mathbf{s}}$.

Exercise 9: For the following quantratic form,

$$S = A_{ij} x_i x_j \quad (i, j = 1,2,3)$$

where A_{ij} are constants, show that,

$$\frac{\partial S}{\partial x_i} = (A_{ij} + A_{ji}) x_j \quad , \quad \frac{\partial^2 S}{\partial x_i \partial x_j} = A_{ij} + A_{ji}$$

Simplify when $A_{ij} = A_{ji}$